

Preparing Middle School Students for the
Transition to High School Mathematics:
Assessing Latinas/os' Mathematical Understanding,
Academic Language and English Language Proficiency

Eduardo Mosqueda
University of California, Santa Cruz

Marco Bravo
Santa Clara University

Jorge Solís
University of Texas, San Antonio

Saúl I. Maldonado
San Diego State University

Jonathan De La Rosa
Santa Clara University

Abstract

Latina/o English Language Learners (ELLs) in mathematics classes have the dual responsibility of understanding mathematics content while simultaneously acquiring disciplinary language in English. The Authentic Math and Writing Assessment System was a formative assessment developed to offer middle school mathematics teachers' instructional guidance for addressing the language and mathematics learning needs of ELLs. Mixed methods analysis of student writing samples focused on mathematical content and revealed possibilities for informing teachers about practices that address both language and content learning goals.

Assessing Latina/os' Mathematical Understanding, Academic Language, and ELP

Introduction

From 2001 to 2011, the number of Latinas/os enrolled in PK-12 public schools increased from 8.2 to 11.8 million students, a shift from 17 to 24 percent of all U.S. students (Kena et al., 2014). In California, an estimated 1.6 million students are identified as English language learners (ELs); of these, 85 percent are Spanish-speakers (DataQuest, 2014)¹. Although more than 450 of the world's languages are represented in U.S. schools, nearly 80% of all ELLs speak Spanish at home, and the overwhelming majority of native Spanish-speakers (approximately 75%) are of Mexican heritage (Téllez, 2010). Spanish-speaking ELLs are often immigrants or the U.S.-born children of immigrants and represent a vast range of language proficiency in both Spanish and English (Mosqueda & Maldonado, 2013). The linguistic characteristics of Latina/o students vary widely; many achieve full English proficiency and exit from specialized language programming, but a substantial number are classified as ELLs, even after several years of instruction in English. This subgroup (Latino/a ELLs) is of particular interest because they tend to face persistent academic challenges.

In addition to making up a significant portion of the school-age population, Latina/o ELLs are also likely to live in poverty, attend schools designated as underperforming, and disproportionately miss out on college-preparatory mathematics coursework (Callahan, 2005; Gándara & Contreras, 2009; Mosqueda, 2010). With respect to mathematics, students' degree of English-language proficiency (ELP) is inappropriately utilized to mediate access to mathematics learning with ELLs overrepresented in low-level mathematics courses (Gándara, Rumberger, Maxwell-Jolly, & Callahan, 2003; Martiniello, 2008; Solano-Flores, 2008). As a result, Latina/o ELLs have consistently scored considerably lower than non-ELL Latina/os on standardized mathematics tests, such as the National Assessment of Educational Progress (Abedi, 2004; Flores, 2007).

Contributing to the underperformance of Latino ELLs in mathematics is the dearth of professional development for teachers to better assist ELLs (Gándara, Maxwell-Jolly & Driscoll 2005). With the implementation of the Common Core State Standards (CCSS) in several states and the pronounced attentiveness to the role of language in teaching and learning mathematics (Lee, Quinn & Valdés, 2013), a teacher's ability to provide linguistic support for ELLs is imperative. For example, expectations set forth by the CCSS for students include constructing viable arguments and critiquing others' reasoning to demonstrate mathematical understanding (Lee et al., 2013). Teachers of ELLs will be required to consider various language domains, including the reading, writing, speaking and listening comprehension needed to meaningfully participate and benefit from such instructional activities. In this paper, we argue that that lack of

¹ We acknowledge here that state and national language classification systems fail to adequately describe the varied and developing academic and language background of students, including students' primary language skills and bilingual abilities.

understanding of the role of English language proficiency in instruction and assessment results in ELs students' lower mathematics achievement owing to the absence of linguistic support strategies in the classroom. The present study explores our efforts in working with teachers of ELLs to identify, assess and provide an instructional toolkit regarding the language necessary to understand and apply mathematics, as outlined by Lee, Quinn and Valdés (2013).

Learning English in the Mathematics Classroom

The teaching and learning of mathematics offers both opportunities and challenges for ELLs, yet limited research has been conducted examining these contexts (Janzen, 2008). On the one hand, the contextualized nature of the subject (e.g., visual representations of concepts, the presence of manipulatives) provides some of the needed language supports for ELLs to fully engage in the subject (Secada, 1996) when given opportunities to learn challenging mathematics curriculum. Yet, the presence of abstract mathematical concepts, mathematical symbols and notations, and unfamiliar ways of using academic language for mathematical reasoning (Echevarria, Richards-Tutor, Chinn & Ratleff, 2011; Lager, 2006; Michaels, O'Connor, & Resnick, 2008; Wong-Fillmore, 2007), present issues for ELLs, unless these learning challenges are addressed by teachers. Several studies have shown promise when classroom and curricular language features receive explicit instructional attention (Abedi & Lord, 2001; Gerena & Keiler, 2012; Lee, Maerten-Rivera, Penfield, LeRoy & Secada, 2008; Martinello, 2008; Snow, Lawrence & White, 2009). For example, nuanced attention to literacy functions, as proposed by the Common Core State Standards (CCSS), extends far beyond merely addressing difficulties with academic vocabulary for ELLs in the content areas. Therefore, teachers of ELLs need to address the language devices used in particular subject areas that frame ideas, show relationships between concepts, and support explanations and arguments that differ between spoken everyday language and written language (Fillmore & Fillmore, 2012).

Lager's (2006) study of middle school ELLs examined the reading challenges related to differences between everyday language and mathematics registers that hinder ELLs when responding to algebraic items in relation to a linear pattern. He examined the transition that many ELLs face in translating and interpreting everyday language to mathematical classroom language. The study involved a sample of 221 middle-school ELLs and non-ELLs in low performing schools including 133 ELLs and 88 Non-ELLs, with a majority of Spanish-speaking Latina/o ELLs representing a range of English language proficiency levels. Participants were asked to answer a set of text-based items related to a linear pattern and asked to construct extended visual representations of a pattern after being given initial figures of that same pattern. Results show that ELLs had more difficulty with these items due to language differences often not associated with mathematics. ELLs reported having most difficulty with understanding words like *extension*, *previous*, and *pattern*. While *pattern* is a word frequently taught explicitly in mathematics and often considered a new mathematics term, *extension* and *previous* are words that are not explicitly addressed in mathematics lessons because they are considered part of the everyday English-language register. Yet, ELLs would have benefited from more explicit attention

to these terms to comprehend mathematical tasks. Other terms like *figure* (i.e. to figure out, figure number) presented multiple related meanings for ELLs to decipher that created breakdowns in comprehension. The study's findings make a strong case for making "explicit the kinds of implicit mathematics-language reading interactions that hinder students" from making sense of written mathematical tasks in classroom lessons and assessments, especially for ELLs (p. 194).

Lee et al., (2008) worked with teachers to implement mathematics instruction in the context of science. Elementary grade teachers received professional development on approaches for addressing the literacy and language involved in doing math and science. Of particular focus was supporting teachers in utilizing ELLs' native language as a resource to access abstract science and mathematics concepts. Results from this study showed treatment group students achieved higher test scores on a statewide mathematics test than the control group. This growth was significant in the area of the statewide assessment that paralleled the topic emphasis of the curriculum. Similar to Lee and her colleagues (2008), we focus on how specific instructional approaches that take into account ELLs' native language can result in improved academic outcomes.

Subjects such as mathematics present ELLs with a series of abstract vocabulary terms and concepts that can be an obstacle to deep mathematical learning. Snow, Lawrence and White (2009) studied "*Word Generation*," a literacy intervention in which ELLs in the treatment group received strong exposure to explicit vocabulary instruction across subject areas. For example, high leverage math terms were used by the teachers and elicited from students in math discussions regarding word problems, with teachers revoicing the correct usage and meaning of the target words. In classrooms where the *Word Generation* intervention was implemented, ELLs showed greater growth on curriculum-specific tests than in comparison classrooms.

Evidence from assessment literature (Abedi & Lord, 2001; Martinello, 2008) illustrates that beyond vocabulary, other linguistic complexities of mathematics can create difficulties in making sense of mathematical concepts for ELLs. Abedi and Lord (2001) administered math test items from the National Assessment of Educational Progress. The test items were modified to reduce their linguistic complexity and modifications included a change of passive to active voice, shortening of long nominalizations and the removal of relative clauses. Interviews with students revealed ELLs' preference for the modified test items, and more importantly, ELLs performed better on the linguistically modified math assessment.

Finally, professional development approaches that make explicit the language of learning in diverse mathematics classrooms by utilizing approaches such as Accountable Talk (Chapin, O'Connor, and Anderson, 2009) indicate that all classroom contexts benefit from the structuring of deliberate talk during mathematics lessons (Michaels, O'Connor, and Resnick, 2008; O'Connor, 2001). Accepting the fact that ELLs represent a diverse group of students, many are not familiar with the classroom norms for participating in dialogic and interactive classroom activities critical for practicing academic language functions. Teachers that orchestrate and

provide models for engaging in explanations and argumentation are able to meaningfully include ELLs in learning contexts.

Formative Assessment for English Learners

The process of assessing content area knowledge and skills of all students, particularly ELLs, is a complex and delicate endeavor. The Standards for Educational and Psychological Testing remind us that every assessment is first and foremost an assessment of language (AERA, APA, NCME, 1999). This statement is further supported by evidence that linguistic modifications of mathematics assessment items can significantly increase results for ELLs (Sato, Rabinowitz, Gallagher, and Huang, 2010). As statewide academic assessments under the CCSS include more open-response items in end-of-year tests, issues of validity and reliability are raised resulting from the process of testing ELLs using exams in English, students' non-dominant language. As such, ELLs' linguistic disadvantages in mathematics achievement are perpetuated and potentially exacerbated.

Recent developments in the use of open-response assessments to gauge multiple dimensions of students' understanding have the potential to be extended to support the development of English-language skills in content area courses such as math for ELLs. Specifically, the formative assessment process can allow educators to account for language development in ELLs as well as measure conceptual understanding in math. Formative assessment involves the gathering of evidence for the purpose of providing feedback about learning as instruction is being delivered (Heritage, 2010). Data gathered via formative assessment is used to make student thinking transparent to teachers, and also to help teachers adapt and improve instructional approaches to more closely support students' learning goals (Ruiz-Primo Furtak, Ayala, Yin, & Shavelson, 2010). Cizek (2010) has shown that formative assessment can be used to effectively measure multiple dimensions of student learning goals such as: identifying students' strengths and weaknesses, helping students guide their own learning, revise and evaluate their work, and develop a sense of autonomy and responsibility for learning.

Formative assessment is typically embedded in an instructional unit and the results must be used to inform the learning goals of a specific lesson (Ruiz-Primo et al., 2010). A critical feature of formative assessment includes the provision of effective feedback to students based on the assessments results (Sadler, 1989). Despite the potential of formative assessment to improve teacher practice, their effective implementation requires a high degree of pedagogical and assessment knowledge. Heritage (2010) has argued that there are four critical dimensions of teacher knowledge that are key to implement formative assessment successfully: 1) domain knowledge, 2) pedagogical content knowledge, 3) knowledge of students' previous learning, and 4) knowledge of assessment. Thus, the lack of training opportunities in assessment for practicing teachers is an issue that cannot be ignored, particularly for teachers of ELLs (Téllez & Mosqueda, 2015).

The literature focused on formative assessment of ELLs is scant (Duran, 2008; Llosa, 2011). The work of Solano-Flores (2006) has examined the linguistic and cultural sources of

measurement error on summative assessments, providing a useful set of criteria to improve the effectiveness of formative assessment for ELLs. The process of adequately assessing ELLs' content knowledge mastery is a complex endeavor given the psychometric limitations not limited to construct-irrelevant variance, or the errors in measurement of subject-matter understanding of ELLs that arise from their low levels of English language proficiency (Abedi, 2004; Duran, 2008). For example, an examination of ELL students' responses on the Massachusetts state exam showed how bias resulting from unnecessary syntactic and lexical complexity in test items contributed to the underperformance of ELLs and resulted in diminished test scores (Martiniello, 2008).

Ultimately, when standardized assessment items are embedded with decontextualized multiple-meaning words, figurative language, metaphors, clichés, and idioms, students must have a high command of English in order to demonstrate their mathematical content knowledge understanding. Wong-Fillmore (2007) describes the linguistic challenges faced by ELLs when examining a mathematics test item from the California High School Exit Exam (CAHSEE) that states, "In probability, an event is a particular happening that may or may not occur." She notes that the word *event* has a specialized meaning, the event is a conditional *particular happening*, and a relative clause modifies *happening* (Wong-Fillmore, 2007, p. 339). As a result, only test takers possessing advanced English language skills are likely to answer the item correctly.

The language demands of the mathematics classroom require substantial understanding of language to not only do math but to explain mathematical reasoning--a major emphasis in the new Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The research we have reviewed suggests that leveraging ELLs' native language along with explicit instructional attention to language in both teaching and assessment settings can mitigate some of these challenges. Our intent with this project was to test the possibility of AMWAS to address this concern.

Setting

The AMWAS activities took place during an intensive mathematics summer program at a charter school in California's Bay Area. The objective of the summer program was to prepare students to face more advanced mathematics courses including college-preparatory courses. The summer program was also conceived as an intervention to support incoming ninth graders transition to high school coursework. Students received after-school tutoring and also participated in field trips to local universities.

Participants

A total of 104 incoming ninth grade students attended summer mathematics classes over seven-weeks, including 46% female students with a majority of students qualifying for the free/reduced lunch program. There were eight mathematics teachers involved in the AMWAS project teaching a range of mathematic courses including Pre-Algebra, Algebra Restart, Algebra,

Geometry, and Algebra II. Participating mathematics teachers had between 5-25 years of teaching experience; three teachers also considered themselves bilingual (English-Spanish) and used their bilingualism in their teaching. As part of the AMWAS Project, mathematics teachers also participated in professional development activities designed to support the integration of language and literacy in mathematics activities as a strategy to augment mathematics content and language development for all students, including ELLs. Eight teachers attended professional development institutes—provided by the authors—before the mathematics summer program started and also met with coaches (also conducted by authors) during the program to co-construct lessons using instructional practices presented in the professional development institutes.

AMWAS (Adolescent Mathematics Writing Assessment System) Design

The primary goal of the **AMWAS** model was to provide teachers with periodic indicators of students' math and literacy abilities and to inform teacher practice. The model involved various stages, including: a) administration, scoring and analyzing student data; b) professional development activities, including customized coaching sessions; and c) augmentation of teacher practice. Figure 1 illustrates the cyclical nature of this model.

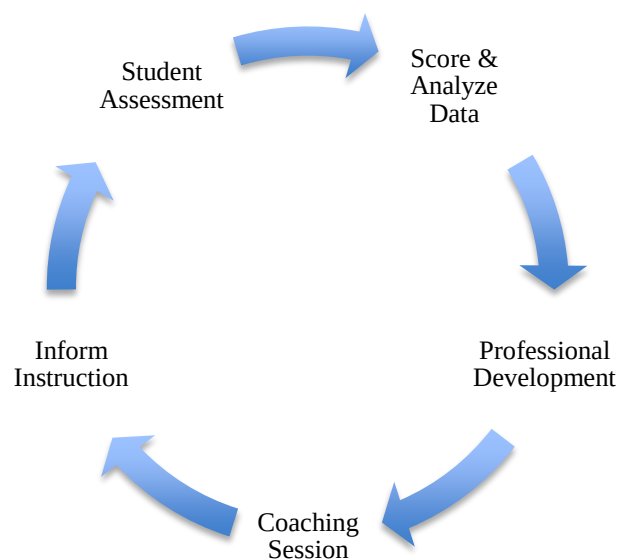
Student Assessment. With teacher input, two writing prompts were developed to gauge students' math reasoning, computation skills, vocabulary use, and conventions. Students were provided with a prompt and asked to respond, with a 30-minute time limit. The initial writing prompt presented a plot graphic of an individual's savings account balance for the year, with the y-axis representing the number of dollars in the savings account (\$0-2000) and the x-axis representing the number of months (1-12). The graphic was introduced as follows:

Directions. **Write at least a paragraph that explains the graph below to someone that is unfamiliar with the graph.** Make sure you use complete sentences and correct punctuation. Use math vocabulary in your response and also explain the math involved in interpreting the graph.

With advice from teachers in the program, a second data set was also presented in a prompt that did not focus on a savings account. The second graphic contained data about an individual's wireless cell phone plan. The y-axis represented dollars (\$0-\$60), while the x-axis represented the number of minutes (0-80).

Score and Analyze Data. Once students completed their response to the prompt, teachers collected the responses and data were given to the researchers. Researchers then calibrated scoring a set of 10 student responses, attaining an 89% inter-rater reliability score.

Figure 1
AMWAS Model



A writing rubric was developed with Math and English Language Development grade-level standards in mind. Scores by classroom were used to inform the nature of the professional development sessions. To capture growth over time, the rubric included 4 domains:

- 1.) Math Reasoning
- 2.) Math Computation
- 3.) Vocabulary Use
- 4.) Conventions

and spanned across four levels:

- 1.) Inaccurate Understanding
- 2.) Incomplete Understanding
- 3.) Meets Expectations
- 4.) Exceeds Expectations.

Professional Development. In the two-hour professional development sessions, teachers scored student responses to gain familiarity with the assessment rubric and to look closely at patterns in student responses to lead to productive conversations about what instructional decisions might be needed to further support student learning. During the three cycles of implementation, the following themes were a focus of the professional development sessions:

1. Reading about Latina/o Mathematicians,
2. Daily Math Writing, and
3. Math Discourse Activities.

The reading sessions involved reading biographies of Latinos that used mathematics in their profession. For example, students read the biography of Jose M. Hernandez, a former astronaut, and how mathematics was a key gateway for him in his profession. Such biographical pieces were selected in response to teacher concerns regarding student motivation. The second set of activities involved providing students more opportunities to practice writing. Teachers instituted

a daily journal writing routine where students responded to such prompts as: “Solve the problem written on the board by your teacher and then explain how you solved it.” In some instances, teachers allowed for peer editing of these writing responses.

To also assist with grammar and vocabulary use, teachers also used daily math language routines as introductory activities. Teachers would write a word problem on the board that contained errors in punctuation, grammar, vocabulary use and spelling and asked students to identify the problems in small groups. Discussion would ensue where teacher would provide the errors in the problem and why they were considered errors. Students then corrected the word problem and then moved on to solve the word problem as well. Teachers used the following procedure to model how to attack solving word problem:

1. Read problem carefully,
2. List facts given,
3. Figure out what the problem is asking for,
4. Eliminate unimportant information (vocabulary & numbers)
5. Draw a diagram,
6. Find and apply appropriate formula,
7. Do the Math, and
8. Check your answer.

Students were then required to identify these pieces as they explained how they solved the problem to the larger class. The Math Discourse activities created situations where students discussed processes for solving problems with the intent of getting students to verbalize their thinking, use math vocabulary and math reasoning to support their claims.

Coaching Session. Following professional development sessions, teacher and researchers co-constructed lessons (1 hour session). In a pre-lesson session, the pair identified learning and language objectives and a sequence to unfold the instruction. When the teacher taught the lesson, the researcher observed the lesson (1.5 hours) taking meticulous notes on the lesson. The two then met after the lesson (30 minutes) to debrief the lesson and identify areas of strength and need, with an understanding that the areas of need would be targeted in subsequent instructional units.

Inform Instruction. Through the various stages mentioned above, it was the goal that such instructional considerations could then become part of the repertoire of teachers, as they provided instruction to students and until the following assessment cycle, where other instructional supports might be needed. This cycle was repeated three times during the summer program, with each cycle serving as an instructional compass for teachers with the goal of addressing students' math learning needs as well as their language learning needs.

Measures and Procedures

Pretest and posttest writing prompts that elicited mathematical understanding and literacy skills were administered by project teachers, roughly 10 weeks apart in summer 2013. Four

project researchers calibrated (reaching an Inter-Rater Reliability score of .89) and scored all the writing products. A qualitative and quantitative view of the data was taken. Below we present overall results of the sample as well as illustrate the diversity of responses with the presentation of three written responses from students.

Findings

We hypothesized that students would increase in their math reasoning skills, improve in their computational responses, apply more academic vocabulary and have fewer grammar, spelling and punctuation errors at the conclusion of the study than at the onset. To test this hypothesis, we analyzed the pre and post-test writing products of students. Table 1 represents the scores across the four domains.

Table 1. Mean Pre- and Post-Scores by Domain

Domain	Pre	Post
Reasoning	2.41 (N=99)	2.59 (N=118)
Computation	2.27 (N=99)	2.28 (N=118)
Vocabulary	2.46 (N=99)	2.42 (N=118)
Conventions	2.33 (N=99)	2.40 (N=118)

Gains were recorded in the *Reasoning* and *Conventions* domains while *Computation and Vocabulary remained relatively constant* from pre to post. Some student responses were written in Spanish and received lower scores.

The Reasoning domain results showed higher positive differences. We found that students did improve from the pre- to post-assessment on mathematical reasoning. The PD sessions emphasized instructional strategies for developing students' mathematical reasoning. The goal of such strategies was to help students articulate their conceptual understanding of mathematical concepts such as linear functions by working in small groups. Students were asked to consider practical applications of linear functions in their daily lives that could be modeled using linear functions. For instance, students were asked to consider annual gym memberships or monthly phone plans that include a base price and a constant fee. Below we highlight two cases that illustrate the most common student errors we observed after scoring students' responses to open-ended prompts.

Discussion

At the initial administration of the writing assessment it was clear that among the students, there was a wide range of not only math understanding but also of writing proficiencies that impeded some students from voicing their comprehension. Some students addressed the writing prompt in Spanish, others illustrated strong conceptual understanding but poor writing conventions and still others had strong writing abilities but lacked conceptual clarity. These wide ranging skills with both language and math concepts was revealing to the mathematics teachers and sparked new practices to address students' individualized needs. In the examples below, we

highlight several student responses to the writing prompt, illustrate the instructional support that was made available to teachers, and unpack the possibilities and limitations of this formative assessment process.

Sergio. In his response, Sergio, an 8th grade ELL in the Algebra class, illustrates several strengths and areas for growth. His writing response was given a score of 2 for *Mathematical Reasoning*, 2 for *Mathematical Computation*, 2 for *Math Vocabulary*, and 1 for *Conventions*. He demonstrated the correct math literacy needed to read the graphic representation, by stating “The number gets bigger and bigger each month...” Yet, the response lacks specificity in terms of actual increases in dollar amounts per month. This level of precision with language is an element of the academic register of math that often eludes students, especially ELLs. Similarly, the absence of actual units, in this case dollars, also signals a need for instructional attention to addressing the nuances of the language of math. Sergio does make mention of “money,” but disassociated from the mention of the amount and again, mentioning the unit as “money,” also lacks the precision called for by the mathematics register. He does remark about the unit of time (month), which can serve as an anchor from which to build understanding by ensuring to include a complete reference of the units involved in the problem.

While Sergio does read the graph correctly, he does not provide adequate reasons as to why there is an increase to the savings account. His response does mention “it incrising [*sic*] by Hundreds...” but does not explain how this increase takes place in relation to time. Referencing the first and last data point in relation to changes in the y and x-axis would have strengthened the reasoning behind the computation involved in reading the graph. Strategies to assist Sergio to zoom in—look at unit change in relation to both x and y axis—as well as to zoom out—look at changes over the entire time span—could assist Sergio in providing a more accurate response. Conditional phrases (if this...then that...) also may have been the linguistic support that he needed to respond to the prompt.

Perhaps part of the trouble Sergio had in providing a response was a lack of vocabulary to explain his mathematical reasoning as he deciphered the graph. A score of 2 on the rubric represents the presence of either Algebra terms (e.g., x-axis, slope, linear function) or math process terms (e.g., calculate, increase). In Sergio’s case, the reference to “increasing” illustrates an element of sophisticated language use. Yet, the lack of algebraic terms could be related to his need for explaining his reasoning behind his interpretation of the graph. To support math vocabulary development, more sustained listening and speaking opportunities with these terms may be needed.

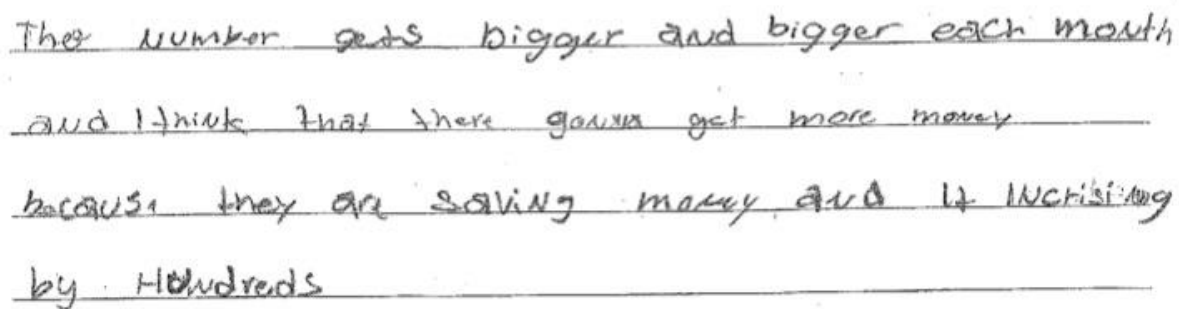
Writing conventions was the most difficult for Sergio. He wrote one long run-on sentence with spelling (e.g., gonna, incrising), capitalization (e.g., Number, Hundreds) and grammar (e.g., **there* instead of *they’re*, **it incrising* instead of *it is increasing*) errors. These errors were telling of the struggles ELLs experience in dealing with the cognitive load involved in the computational and reasoning aspects of mathematical tasks, but also the linguistic load in explaining their thinking either through talk or in written form. Notwithstanding, it is through

this process of verbalizing his thinking that the conceptual understanding is made more enduring for Sergio.

Temo. This last example illustrates once again the range of academic and language resources incoming 9th grade ELL students have for writing in mathematics (see Figure 3).

Figure 2

Sergio's Assessment Response



The number gets bigger and bigger each month
and I think that there gonna get more money
because they are saving money and it increasing
by Hundreds

Temo's writing response was given a score of 4 for *Mathematical Reasoning*, 3 for *Mathematical Computation*, 2 for *Math Vocabulary*, and 1 for *Conventions*. Moreover, this example demonstrates the viable potential for drawing from ELLs primary language to develop mathematical reasoning and disciplinary language functions. While the instructions of the assessment were written in English and the assessment protocol provided minimal oral language support in any language, students were given the option to write a response in any language.

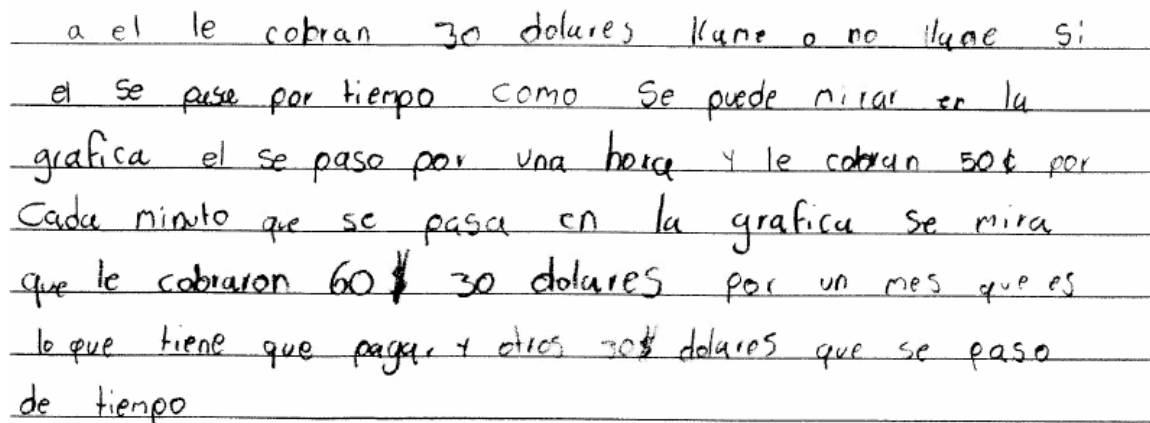
Temo interpreted the question and responded entirely in Spanish. Temo is considered a beginning level English language[1] learner based on the state English proficiency assessment, and he has been in the U.S. less than three years. In terms of the mathematical reasoning domain score, he exceeds grade level expectations by identifying key components associated with linear functions including the y intercept value where he states that the user is charged 30 dollars if he calls or doesn't call someone ("a el le cobran 30 dolares llame o no llame"). This information is presented in the writing prompt where the user has a "\$30 base price and a per minute charge." Temo also addresses the relationship between the "x" and "y" values on the graph related to cost of usage by identifying the relevant money units like dollars and cents ("dolares," "50¢," "60\$") in relation to time of usage like month, per time used, and for one hour ("mes,"

“por tiempo,” “una hora”). Of note here is that Temo clearly understands the linear pattern associated with phone usage and cost. He identifies the variable cost per minute at 50 cents per minute (“le cobran 50¢ por cada minuto que pasa”) and the cost for 60 additional minutes (“se mira que le cobran 60\$”) and the fixed monthly cost at \$30 (“30 dolares por un mes que es lo que tiene que pagar”). This explanation reinforces Temo’s understanding once again that it costs 50 cents per minute which is the slope.

Temo produces a sound mathematical explanation using precise figures to denote his understanding of the problem while not using an explicit formula to solve it such as $y = mx + b$ and related concepts. In this regard however his mathematical computation meets grade level expectations because he accurately solves the problem. In doing so, Temo successfully uses key money symbols (“50¢”, “30 dolares”, “60\$”) used to express values per unit and academic terms associated with mathematical operations such as for every minute (“50¢ por cada minuto”) and “for one month” (“30 dolares por un mes”). These examples demonstrate that Temo is familiar with some mathematical terminology under the domain of academic words. These terms can be the most challenging types of academic vocabulary for ELLs as they present multiple ways of interpretation across the disciplines and for specific purposes. However, we don’t see evidence that Temo is more familiar with technical mathematical terms related to this problem

Figure 3

Temo’s Assessment Response



a el le cobran 30 dolares kune o no kune si
 el se pase por tiempo como se puede mirar en la
 grafica el se paso por una hora y le cobran 50¢ por
 Cada minuto que se pasa en la grafica se mira
 que le cobraron 60\$ 30 dolares por un mes que es
 lo que tiene que pagar y otros 30\$ dolares que se paso
 de tiempo

such as slope (“pendiente”), linear (“lineal” or “función lineal”), or y-intercept (“ordenada al origen”, “corte con el eje y”). Finally, there appears to be some transposition in the expression of money between both languages where Temo expresses the dollar sign in front of the value such as “60\$” and again with “30\$ dolares”, an informal usage of the money symbol for pesos. This occurrence supports the view that Temo might have had interruptions in formal schooling and potentially limited practice using these symbols.

Finally, Temo does not use basic standard writing conventions common in both languages (i.e. periods, commas, capitalization) which further suggests limited writing support not

commensurate with an 8th grade student. Temo also does not use any accents in his spelling of Spanish words (i.e., “dólares”, “la gráfica”, “se pasó”) which again is a standard Spanish language convention learned early in schooling. These omissions are significant because these writing conventions are used in all subject areas. That is, Temo has not been able to transfer the use of basic writing conventions that he has been exposed to in English and Spanish for writing or for writing for mathematical purposes. Despite Temo’s lapses in writing conventions, he is able to correctly interpret the writing task, make sense of it, and produce a comprehensible response while traversing two languages. Temo’s writing example provides a particular insight into the often-dichotomous nature of writing in mathematics for ELLs.

Quantitative results of this study illustrate particular trends—vocabulary and math computation may require more instructional attention than math reasoning and writing conventions. Yet within these trends, students like Sergio and Temo paint very diverse pictures of the intertwined nature of language and mathematics. Both data sets helped inform the professional development teachers received and subsequent instructional attention teachers provided in their math classrooms.

Conclusion

The formative assessment process and PD that was included in AMWAS allowed us to gauge students’ mathematical reasoning, conceptual understanding, vocabulary use and writing conventions by providing a clearer picture of how English-language and literacy abilities impact students’ ability to explain their reasoning and conceptual understanding. Temo, for example, who was allowed to write his response in Spanish was not only able to articulate his reasoning for his explanation of the graph, but did so with the use of appropriate and sophisticated math terms. Sergio, on the other hand, who was ‘between languages,’ was able to write a response that illustrated some conceptual clarity, but writing conventions in English may have limited his ability to fully explain his reasoning. Such diverse outcomes would not have been made as visible for the classroom teacher without the use of formative assessment that captured biliteracy skills in mathematics. Trends among students within each class informed coaching cycles, which in turn allowed researchers/coaches to assist teachers in designing, delivering and assessing math instruction that attended to math learning goals and specific language learning goals to allow students an opportunity to communicate the depth of their mathematical thinking.

References

- Abedi, J. (2004). The No Child Left Behind Act and English language learners: Assessment and accountability issues. *Educational Researcher*. Vol. 33, No 1, 4-14.
- Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education*, 14, 219- 234.
- American Educational Research Association, American Psychological Association, & National Council on Measurement in Education. (1999). *Standards for educational and psychological testing*. Washington, DC: American Psychological Association.
- Callahan, R. (2005). Tracking and high school English learners: Limiting opportunity to learn. *American Educational Research Journal*, 42, 305–328.
- Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom*

- discussions: Using math talk to help students learn, grades K-6*. Math Solutions.
- Cizek, G. J. (2010). An introduction to formative assessment. H. Andrade & G. Cizek (Eds.). *Handbook of formative assessment*. New York: Routledge.
- Duran, R. P. (2008). Assessing English language learners' achievement. *Review of Research in Education*, 32(1), 292.
- Echevarria, J., Richards-Tutor, C., Chinn, V. P., & Ratleff, A. R. (2011). Did they get it? The role of fidelity in teaching English learners. *Journal of Adolescent & Adult Literacy*, 54, 425-234.
- Fillmore, L. W., & Fillmore, C. J. (2012). What Does Text Complexity Mean for English Learners and Language Minority Students?. *Commissioned Papers on Language and Literacy Issues in the Common Core State Standards and Next Generation Science Standards*, 94, 64.
- Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap? *The High School Journal*, 91(1), 29-42.
- Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Cambridge, MA: Harvard University Press.
- Gándara, P., Rumberger, R. W., Maxwell-Jolly, J., & Callahan, R. (2003). English Learners in California schools: Unequal resources, unequal outcomes. *Educational Policy Analysis Archives*, 11. Retrieved September 1, 2015 from <http://epaa.asu.edu/epaa/v11n36/>.
- Gándara, P., Maxwell-Jolly, J. & Driscoll, A. (2005). Listening to Teachers of ELLs: A Survey of California Teachers' Challenges, Experiences, and Professional Development Needs. Policy Analysis for California Education, PACE.
- Gerena, L., & Keiler, L. (2012). Effective intervention with urban secondary English language learners: How peer instructors support learning. *Bilingual Research Journal*, 35(1), 76-97.
- Heritage, M. (2010). Formative assessment and next-generation assessment systems: Are we losing an opportunity?. *Council of Chief State School Officers*.
- Heubert, J. P., & Hauser, R. M. (1999). *High stakes: Testing for tracking, promotion, and graduation*. Washington, DC: National Research Council, National Academy Press.
- Janzen, J. (2008). Teaching English language learners in the content areas. *Review of Educational Research*, 78(4), 1010-1038.
- Kena, G., Aud, S., Johnson, F., Wang, X., Zhang, J., Rathbun, A., ... & Kristapovich, P. (2014). The Condition of Education 2014. NCES 2014-083. *National Center for Education Statistics*.
- Lager, C. A. (2006). Types of mathematics-language reading interactions that unnecessarily hinder algebra learning and assessment. *Reading Psychology*, 27(2-3), 165-204.
- Lee, O., Quinn, H. & Valdés, G. (2013). Science and Language for English Language Learners in Relation to Next Generation Science Standards and with Implications for Common Core State Standards for English Language Arts and Mathematics. *Educational Researcher*, 43, 223-233.
- Lee, O., Maerten-Rivera, J., Penfield, R. D., LeRoy, K. & Secada (2008). Science Achievement of ELLs in Urban Elementary Schools: Results of a First-Year Professional Development Intervention. *Journal of Research in Science Teaching*, 45(1), 31-52.
- Llosa, L. (2011). Standards-based classroom assessment of English proficiency: A review

- of issues, current developments, and future directions for research. *Language Testing*, 28(3), 367-382.
- Martiniello, M. (2008). Language and the performance of English language learners in math word problems. *Harvard Educational Review*, 78(2), 333-368.
- Michaels, S., O'Connor, C., & Resnick, L. B. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education*, 27(4), 283-297.
- Mosqueda, E. (2010). Compounding Inequalities: English proficiency and tracking and their relation to mathematics performance among Latina/o secondary school youth. *Journal of Urban Mathematics Education*, 3(1), pp. 57-81.
- Mosqueda, E. & Maldonado, S. I. (2013). The effects of English language proficiency and curricular pathways: Latina/os' mathematics achievement in secondary schools. *Equity & Excellence in Education* 46(2), pp. 202 – 219.
- National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core state Standards for English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects. Washington DC: Authors.
- O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?" A case study of a mathematical group discussion. *Educational Studies in Mathematics*, 46(1-3), 143-185.
- Ruiz-Primo, M. A., Furtak, E. M., Ayala, C., Yin, Y., & Shavelson, R. J. (2010). *Formative assessment, motivation, and science learning*. H. Andrade & G. Cizek (Eds.) (2010). Handbook of formative assessment. New York: Routledge.
- Sadler, D. R. (1989). Formative assessment and the design of instructional systems. *Instructional Science*, 18, 119-144.
- Sato, E., Rabinowitz, S., Gallagher, C., & Huang, C. W. (2010). Accommodations for English Language Learner Students: The Effect of Linguistic Modification of Math Test Item Sets. Final Report. NCEE 2009-4079. *National Center for Education Evaluation and Regional Assistance*.
- Secada, W.G. (1996). Urban students acquiring English and learning mathematics in the context of reform. *Urban Education*, 30(4) (pp.422-448).
- Solano-Flores, G. (2006). Language, dialect, and register: Sociolinguistics and the estimation of measurement error in the testing of English-language learners. *Teachers College Record*, 108(11), 2354–2379.
- Solano-Flores, G. (2008). Who is given tests in what language by whom, when, and where? The need for probabilistic views of language in the testing of English language learners. *Educational Researcher*, 37(4), 189-199.
- Snow, C., Lawrence, J., & White, C. (2009). Generating knowledge of academic language among urban middle school students. *Journal of Research on Educational Effectiveness*, 2(4), 325–344.
- Téllez, K. (2010). *Teaching English language learners: Fostering language and the democratic experience*. Boulder, CO: Paradigm Publishers.
- Tellez, K. & Mosqueda, E. (2015). Developing teachers' knowledge and skills at the intersection of language learners and language assessment. *Review of Research in Education*, 39, 87 – 121.
- Wong-Fillmore, L. (2007). English learners and mathematical learning: Language issues to consider. *Assessing Mathematical Proficiency*, 53, 333-344.

Appendices

Appendix 1

AMWAS Administration Guidelines

Adolescent Mathematics and Writing Assessment System (AMWAS)

Administration. The AMWAS instrument is designed to assess students' math conceptual understanding and writing skills. The AMWAS assessment gauges students' writing in 4 domains:

- a. Vocabulary use
- b. Math Conceptual Accuracy
- c. Grammar
- d. Conventions

Follow the following steps when administering the assessment.

1. Before administering the AMWAS, familiarize yourself with the writing prompt and make copies of the writing prompt. Also, have pencils available for students to use.
2. Before providing students with the writing prompt, explain to students that for the next 30 minutes, they will have time to respond to the writing prompt. Tell students they must do their own work and that it should be quiet during the time of the assessment.
3. Hand out the writing prompt paper and have students write their name and date on the top of the paper.
4. Read the prompt aloud to students *"Write at least a paragraph explaining the graph below to someone that is unfamiliar with the graph. Make sure you use complete sentences and correct punctuation. Use math vocabulary in your response."*
5. Tell students that they should write in English as much as possible, but if they don't know a word in English, they can write it in Spanish.
6. Allow students to read or work on homework if they get done early. Yet, before they are allowed to work on homework or read, have them review their writing before they turn in their final work.
7. Make sure students do their own work and treat the writing assignment as an assessment. Allow students more time if needed. This assessment will be administered three times during the program (Pre, Mid, Post). Results will be shared with teachers to inform instruction.

Appendix 2
Initial Assessment

Adolescent Mathematics and Writing Assessment System (AMWAS)

NAME _____

DATE _____

Directions. Write at least a paragraph that explains the graph below to someone that is **unfamiliar with the graph**. Make sure you use complete sentences and correct punctuation. Use math vocabulary in your response.



Appendix 3
Second Assessment

Adolescent Mathematics Writing Assessment System (AMWAS)

NAME _____ DATE _____ Teacher _____

Directions. Write at least a paragraph that explains the graph below to someone that is unfamiliar with the graph. Saul's wireless plan has a \$30 base price and a per minute charge. Make sure you use complete sentences and correct punctuation. Use math vocabulary in your response and also explain the math involved in interpreting the graph.

